Reactor as a Source of Antineutrinos: Thermal Fission Energy

V. Kopeikin*, L. Mikaelyan†, V. Sinev‡

Russian Research Centre "Kurchatov Institute", Moscow

Abstract

Deeper insight into the features of a reactor as a source of antineutrinos is required for making further advances in studying the fundamental properties of the neutrino. The relationship between the thermal power of a reactor and the rate of the chain fission reaction in its core is analyzed.

Introduction

Experiments aimed at studying the fundamental properties of the neutrino and at testing the standard model of electroweak interactions are being performed at reactors. A collaboration of researchers from the Kurchatov Institute and the Petersburg Nuclear Physics Institute (PNPI, Gatchina) are conducting an experiment devoted to searches for the neutrino anomalous magnetic moment [1]. A group of physicists from the Institute of Theoretical and Experimental Physics (ITEP, Moscow) and the Joint Institute for Nuclear Research (JINR, Dubna) are preparing a similar experiment at the reactor of the Kalinin atomic power plant [2]. The CHOOZ experiment [3], completed quite recently, set constraints on the neutrino-mixing-matrix element U_{e3}. The KamLAND Collaboration, which is recording antineutrinos at a distance of a few hundred kilometers from reactors, is able to determine

*kopeykin@polyn.kiae.su
†mikaelyan@polyn.kiae.su
‡sinev@polyn.kiae.su

the remaining two mixing-matrix elements U_{e1} and U_{e2} and to test the LMA MSW hypothesis of solar-neutrino oscillations [4]. In addition, it should be noted that a program of neutrino studies at the reactors on the Taiwan island is being developed [5] and that interesting proposals concerning searches for neutrino oscillations were put forth in Germany [6]. (More details on the motivation of those investigations, their status, and their prospects can be found, for example, in the review articles cited in [7].)

Differing in many respects, the aforementioned experiments possess one common feature: the results obtained in these experiments are analyzed by an absolute method − specifically, the measured counting rates for neutrino events and their spectral distributions are contrasted against their counterparts calculated on the basis of the theory of electroweak interactions. For input data in these calculations, use is made of the set of features of neutrino radiation that, together with other data, form a metrological basis of the experimental physics of neutrinos at nuclear reactors.

The spectral density $f(E_\nu)$ ($\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{MeV}^{-1}$) of the flux of reactor electron antineutrinos ($\bar{\nu}_e$) incident on a detector is given by

$$f(E_\nu) = N_f \rho_f(E_\nu)/4\pi R^2, \tag{1}$$

where N_f is the number of fission events in a reactor per second, $\rho_f(E_\nu)$ ($\mathrm{MeV}^{-1}\mathrm{fiss.}^{-1}$) is the spectrum of reactor electron antineutrinos that is normalized to a fission event, and R (cm) is the distance between the reactor and the detector used.

In the fission of uranium and plutonium nuclei and in the subsequent radioactive decay of fission fragments, as well as in accompanying neutron reactions, energy is released, its major part being absorbed in the reactor, whereby it is converted into heat. Denoting by E_f (MeV/fiss.) the energy absorbed in the reactor on average per fission event, we can represent the chain-reaction rate N_f in the form

$$N_f = W/E_f, \tag{2}$$

The present study is devoted to exploring the quantity E_f , which relates the fission-reaction rate N_f (fiss./s) to the thermal power W of a reactor. First of all, we consider this relationship for the example of a standard operating period of a reactor belonging to the PWR type, in which case isotopes undergoing fission include ^{235}U, ^{239}Pu, ^{238}U, and ^{241}Pu. The method developed here and, upon introducing some specific corrections, the results

presented below can be used in neutrino experiments, both those that are being presently performed and those that are planned, at reactors of any other type.

The reactor staff determines the current value of the thermal power to a precision of about 1 to 2%. In order to avoid increasing the error in determining the ratio in (2), we will try to calculate the energy E_f to a higher precision.

1 COMPONENTS OF THE ENERGY E_f

The energy E_f can be represented as the sum of four terms; that is,

$$E_f = E_{tot} - \langle E_\nu \rangle - \Delta E_{\beta\gamma} + E_{nc}, \tag{3}$$

where E_{tot} is the total energy released in nuclear fission from the instant at which the neutron inducing this fission process is absorbed to the completion of the beta decays of product fragments and their transformation into beta-stable neutral atoms, $\langle E_\nu \rangle$ is the mean energy carried away by the antineutrinos that are produced in the beta decay of fission fragments ($\sim 6\bar{\nu}_e/fiss.$), $\Delta E_{\beta\gamma}$ is the energy of beta electrons and photons from fission fragments that did not decay at a given instant of time, and E_{nc} is the energy absorbed upon neutron capture (without fission) in various materials of the reactor core.

That part of the total energy E_{tot} which remains in the reactor and which transforms into heat forms the effective fission energy E_{eff},

$$E_{eff} = E_{tot} - \langle E_\nu \rangle - \Delta E_{\beta\gamma}. \tag{3.1}$$

The expression for E_{eff} can then be represented in the form

$$E_f = E_{eff} + E_{nc}. \tag{3.2}$$

The above concerns the energy released in a single nuclear-fission event, but the chain fission reaction in a reactor proceeds over a finite time interval. For this reason, we consider a chain fission reaction that begins at the instant $t = 0$ and proceeds at the rate of $N_f = 1$ fiss./s. We denote by $E(t)_{tot}$ the

energy released per second at the instant t reckoned from the commencement of the process being considered. The quantity $E(t)_{tot}$ includes all kinds of energy, with the exception of E_{nc}, which is the energy that is released in various materials upon the absorption in them of neutrons not involved in the fission process. We will now consider the function $f_{tot}(t)$ determining the energy released per unit time after the lapse of the time t since a single fission event. It is obvious that

$$E(t)_{tot} = \int_0^t f_{tot}(t')dt', \qquad f_{tot}(t) = \frac{dE(t)_{tot}}{dt} \tag{4}$$

The energy $E(t)_{tot}$ grows with increasing fission process time t, tending to the limiting value $E(t)_{\infty}$,

$$E(\infty)_{tot} = \int_0^{\infty} f_{tot}(x)dx \equiv E_{tot}. \tag{4.1}$$

The above equations relate the energy release in a single fission event to the energy release per unit time in a continuous process.

Neutrino investigations are performed at reactors where use is made of uranium whose enrichment in ^{235}U is low. As this isotope burns out, ^{239}Pu and ^{241}Pu are accumulated in the core of such a reactor. Just like ^{235}U, these isotopes undergo fission induced by thermal neutrons. There is also a contribution to the total number of fission events from ^{238}U, which is fissile under the effect of fast neutrons. Therefore, we have

$$E_f = \sum \alpha_i E_{fi}, \qquad \sum \alpha_i = 1, \tag{5}$$

where α_i $(i = 5, 9, 8, 1)-$ are the contributions of the ^{235}U, ^{239}Pu, ^{238}U and ^{241}Pu isotopes to the total number N_f of fission events at a given instant of time. Information about α_i values, which change in the course of reactor operation, is provided by the reactor staff with a relative error of 5%. The α_i values typical of PWR reactors are

$$\alpha_5 = 0.59, \qquad \alpha_9 = 0.29, \qquad \alpha_8 = 0.07, \qquad \alpha_1 = 0.05. \tag{6}$$

It should be emphasized that the energy E_f and the calculated number W/E_f of fission events occurring in a reactor at a given instant of time are not determined exclusively by the current reactor state, which is specified by

the level of the reactor power and by the isotopic composition of the burning nuclear fuel, but they are dependent on the prehistory of the reactor. This dependence is controlled by the terms E_{nc} and $\Delta E_{\beta\gamma}$ which appear in (3). The quantity E_{nc} changes along with the composition of the materials in the reactor core in the course of reactor operation. Both terms involve a contribution from longlived beta emitters and depend on the duration of the irradiation of the fuel.

For the isotopes undergoing fission, the energies E_{fi} in (5) for the whole reactor exceed 200 MeV/fiss. Going somewhat ahead, we note that, for a PWR reactor, the absolute values of the terms appearing in expression (3) for the energy E_f are in the following ratio:

$$E_{tot} : \langle E_\nu \rangle : \Delta E_{\beta\gamma} : E_{nc} \approx 200 : 9 : 0.3 : 10\,. \tag{7}$$

2 TOTAL (E_{tot}) AND EFFECTIVE (E_{eff}) FISSION ENERGY

2.1 *Total Fission Energy E_{tot}*

The energy E_{tot} can be calculated by summing the mean values of various components of the energy release, such as the fragment kinetic energy, the energy of prompt and delayed fission gamma rays, and the neutron and beta-electron kinetic energies. However, much more precise results are obtained by directly applying the energy-conservation law to the fission process; that is,

$$M(A_0, Z_0) + M_n = \sum y_A\, M(A, Z_A) + n_f\, M_n + E_{tot}, \tag{8}$$

where $M(A_0, Z_0)$ is the atomic mass of the isotope undergoing fission (the speed of light is set to unity, c = 1); A_0 and Z_0 are its mass and charge numbers, respectively; M_n is the neutron mass; summation is performed over the mass numbers A of beta-stable fission products; $M(A, Z_A)$ are the masses of these products; y_A are their total yields, $\sum y_A = 2$; and n_f is the mean total number of prompt and delayed fission neutrons (for the obvious reason, the notation ν, which is usually used for the mean number of fission neutrons, is replaced here by n_f.)

Using the condition requiring that the number of nucleons be conserved in the fission process and introducing the mass excesses for atoms, $m(A,Z)$, we can recast relation (8) into form

$$E_{tot} = m(A_0, Z_0) - \sum y_A \, m(A, Z_A) - (n_f - 1)\, m_n \tag{9}$$

where $m(A,Z) = M(A,Z) - A\, m_0$ (m_0 is an atomic mass unit) and $m_n = M_n - m_0 = 8.0713 \pm 0.0001$ MeV is the neutron mass excess.

The calculated total energy E_{tot} and the quantities appearing in relation (9) are given in Table 1 for all four nuclei undergoing fission. In computing these results, we employed data on the mass excesses for the atoms involved [8] and on the yields of fission fragments [9] whose mass numbers took values in the range between 66 and 172 (see Fig. 1). The data on the number of fission neutrons were borrowed from [10].

Table 1: Mass excesses and total fission energy E_{tot} (in MeV/fiss.)

Fissile nucleus	Mass excess $m(A_0, Z_0)$	Mass excess for fission products, $\sum y_A \, m(A, Z_A)$	Number of fission neutrons, n_f	$(n_f - 1) m_n$	Total fission energy, E_{tot}
^{235}U	40.914 $\pm$0.002	-173.43 $\pm$0.05	2.432 $\pm$0.0036	11.55 $\pm$0.03	202.79 $\pm$0.06
^{238}U	47.304 $\pm$0.002	-173.39 $\pm$0.10	2.829 $\pm$0.011	14.76 $\pm$0.09	205.93 $\pm$0.13
^{239}Pu	48.584 $\pm$0.002	-173.87 $\pm$0.07	2.875 $\pm$0.0060	15.13 $\pm$0.05	207.32 $\pm$0.08
^{241}Pu	52.951 $\pm$0.002	-173.72 $\pm$0.10	2.937 $\pm$0.0073	15.63 $\pm$0.06	211.04 $\pm$0.12

For the fissile nuclei being considered, the values of E_{tot} differ from one another by a few MeV, increasing in the order of their positions in the first column of Table 1. These distinctions are caused, above all, by an increase in the mass excess for the atoms of the fissile isotopes and, to a lesser extent, by an increase in the number n_f of fission neutrons. At the same time, it can be seen from Table 1 that, for the set of stable fission fragments, the total mass excess $\sum y_A \, m(A, Z_A)$ is virtually independent of the nucleus undergoing fission. This is because the quantity $m(A, Z_A)$ is approximately constant over

the region of high fragment yields y_A, sizably increasing only for products originating from highly asymmetric fission, where the yields in question are relatively low (see Fig. 1). Therefore, even significant distinctions between the mass distributions of fragments produced in the fission of uranium and plutonium nuclei have but a slight effect on the sums $\sum y_A\, m(A, Z_A)$.

The error in the mass excess $\sum y_A m(A, Z_A)$ (see the third column in Table 1) depends on the uncertainty in the yields y_A, since the overwhelming majority of the values of $m(A, Z_A)$ are known to a precision not poorer than 5 keV. In order to find this error, each of the yields y_A was varied individually, irrespective of the others, under the assumption that it obeys the a Gaussian distribution. Upon each variation, there arises a new set of y_A values, and we calculated the value of $\sum y_A m(A, Z_A)$ for this set. As a result, the total number $\sum y_A A$ of nucleons contained in fission products changed somewhat. On the basis of the relation

$$A_0 + 1 = \sum y_A A + n_f, \tag{10}$$

which expresses the law of nucleon-number conservation, we calculated the corresponding number n_f of neutrons. A point in the plane spanned by the variables $\sum y_A m(A, Z_A)$ and n_f was associated with the pair of values found in this way for the mass excess and the number of neutrons. The results of one such computational experiment performed for ^{235}U, where use was made of a Gaussian distribution characterized by a FWHM value of 0.12, are illustrated in Fig. 2 (ten thousand points). From Fig. 2, it can be seen that the uncertainties in the yields of fission products introduce an error of about 35 keV in the mass excess and that available experimental data on the yields of fission products and on the number of neutrons are quite consistent.

That the calculation of the total energies E_{tot} on the basis of applying the energy-conservation law to the fission process was highly precise was due to the above features.

We also note that, in fact, the quantity $\sum y_A m(A, Z_A)$ is independent of the incident-neutron energy until the yields y_A change significantly near the humps of the mass distributions. The calculations reveal that, in ^{235}U and ^{235}Pu fission induced by neutrons of the fission spectrum, the deviation of $\sum y_A m(A, Z_A)$ from the values presented in Table 1 does not exceed 0.1 MeV.

The values of E_{tot} were obtained without taking into account ternary fission. Ternary fission accompanied by the emission of a long-range alpha particle occurs approximately in one of 500 cases; other types of ternary

fission are much less probable. According to estimates, the change in E_{tot} upon taking into account ternary fission does not exceed 0.02%.

In calculating E_{tot}, we disregarded the alpha decays of ^{144}Nd, ^{147}Sm and ^{149}Sm nuclei, which are formed upon the completion of beta-decay processes. The total yield of these alpha-particle emitters is about 10%; however, their half-lives exceed 10^{11} yr, so that they make no significant contribution to the energy release.

2.2 *Effective Energy* E_{eff}

In this subsection, we describe schematically a procedure for calculating the energies $\langle E_\nu \rangle$ carried away by antineutrinos and the corrections $\Delta E_{\beta\gamma}$ and present the results of these calculations, along with the values found for the effective energies E_{eff} according to relation (3.1).

1. Along with electron antineutrinos ($\bar{\nu}_e$) emitted by fission fragments, a considerable number of electron antineutrinos are generated in a reactor that are emitted in the beta decay of nuclei produced upon the activation of the materials occurring in the reactor by neutrons. In calculating the energy $\langle E_\nu \rangle$, we take into account only those reactor antineutrinos that are emitted by fission fragment not perturbed by the interaction with neutrons rather than all of them.

The $\bar{\nu}_e$ spectrum decreases fast with increasing energy E_ν, virtually vanishing at $E_\nu \approx 10$ MeV. In this spectrum, the hard section $E_\nu \geq 2$ MeV contains about 60% of the energy $\langle E_\nu \rangle$ that is carried away by antineutrinos.

In the case of ^{235}U, ^{239}Pu, and ^{241}Pu, the $\bar{\nu}_e$ spectra necessary for calculating $\langle E_\nu \rangle$ were determined in the following way:

- For the region of energies above 1.8 MeV, use was made of the spectra found in the Laue−Langevin Institute (ILL) by reconstructing the measured spectra of beta electrons emitted by fission fragments [11], small corrections of about 2.5% that correspond to the contributions of long-lived beta emitters [12] and which were disregarded in [11] being introduced in these spectra.

- The $\bar{\nu}_e$ spectra that we calculated for the energy range 0−3 MeV were smoothly matched in the segment between 2 and 2.5 MeV with the corrected ILL spectra. As a result, the calculated values changed by 2 to 3%.

In the case of ^{238}U, the energy $\langle E_\nu \rangle$ was found on the basis of the $\bar{\nu}_e$ spectrum calculated in the present study.

In the region $E_\nu < 2$ MeV, it is not easy to estimate the error in the energy carried away by electron antineutrinos. The database used in the relevant calculation includes information about 571 fission fragments. For them, the overwhelming majority of decay diagrams is well known. The error in determining this part of $\langle E_\nu \rangle$ is likely to be within 4%.

We recall that, in fission, nuclei emit about 6 $\bar{\nu}_e$ of mean energy approximately equal to 1.5 MeV. For the fissile isotopes in question, the $\langle E_\nu \rangle$ values (in MeV/fiss.) found in the way outlined above are

$$\begin{array}{ll} ^{235}\mathrm{U} : 9.07 \pm 0.32 & ^{238}\mathrm{U} : 11.00 \pm 0.80 \\ ^{239}\mathrm{Pu} : 7.22 \pm 0.27 & ^{241}\mathrm{Pu} : 8.71 \pm 0.30 \end{array} \tag{11}$$

We note that the errors in our knowledge of the outgoing-neutrino energies $\langle E_\nu \rangle$ are much greater than the errors in determining the energies E_{tot}.

Part of the energy carried away by antineutrinos of energy $E_\nu \geq 1.8$ MeV can be directly compared with data obtained in an experiment at the reactor of the Rovno atomic power plant [13]. In that experiment, the positron spectrum was measured in the inverse beta-decay reaction $\bar{\nu}_e + p \to n + e^+$ and the $\bar{\nu}_e$ spectrum was reconstructed in the energy region $E_\nu > 1.8$ MeV. The value found with the aid of this spectrum for the energy that is carried away is in satisfactory agreement with that which was calculated in the present study; that is,

$$X_{Rovno/calc} = 4.679/4.815 = 0.972. \tag{12}$$

2. We recall that the energy $E_{\beta\gamma}$ released upon the complete beta decay of a pair of fission fragments is contained in the total fission energy E_{tot}. The correction $\Delta E_{\beta\gamma}(t)$ takes into account the fact that, at the instant of observation t, the decay processes have not yet been completed,

$$\Delta E_{\beta\gamma}(t) = E_{\beta\gamma}(\infty) - E_{\beta\gamma}(t) = \int_t^\infty dt' f_{\beta\gamma}(t'), \tag{13}$$

where $E_{\beta\gamma}(t)$ is the energy released per second at the instant t reckoned from the beginning of the fission process proceeding at a rate of 1 fiss./s and $f_{\beta\gamma}(t)$

is the energy released per unit time after a lapse of time t from a single fission event [compare with the analogous expressions in (4) for E_{tot}].

The energy $\Delta E_{\beta\gamma}(t)$ of fission fragments that did not decay first decreases fast with increasing duration of the irradiation of the fuel used; this decrease gradually becomes slower, with the result that, at irradiation times of about 1.5 yr, $\Delta E_{\beta\gamma}(t)$ virtually reaches a plateau (see Fig. 3). The formation of this plateau is associated with fragments whose lifetime exceeds 30 yr. Presented immediately below are the values of $\Delta E_{\beta\gamma}(t)$ (in MeV/fiss.) at the fuel-irradiation time corresponding to the midpoint of the standard operating period of a PWR reactor:

$$
\begin{aligned}
&{}^{235}\mathrm{U} : 0.35 \pm 0.02 \qquad {}^{238}\mathrm{U} : 0.33 \pm 0.03 \\
&{}^{239}\mathrm{Pu} : 0.30 \pm 0.02 \qquad {}^{241}\mathrm{Pu} : 0.29 \pm 0.03.
\end{aligned}
\tag{14}
$$

It is useful to have an analytic expression for the energy $\Delta E_{\beta\gamma}(t)$. Over a wide interval of the times t, the expression

$$
\begin{aligned}
&{}^{fit}\Delta E_{\beta\gamma}(t) = E_0\, exp(-\lambda_0 t^{\alpha}) + \varepsilon, \\
&0.5 < t < 500\ days
\end{aligned}
\tag{15}
$$

at the E_0, λ_0, α, and ε values given in Table 2 agree with the results of the precise calculation to within 2 %.

Table 2: Parameters of the functions ${}^{fit}\Delta E_{\beta\gamma}(t)$

Fissile nucleus	E_0,MeV	λ_0	α	ε, MeV
^{235}U	8.80	2.15	0.108	0.185
^{238}U	9.20	2.22	0.106	0.165
^{239}Pu	8.50	2.18	0.109	0.155
^{241}Pu	8.20	2.16	0.105	0.135

The first term in (15) describes an exponential decay with a decay probability decreasing with time, while the second term corresponds to the plateau.

3. To conclude this section, we present the values of the effective fission energy E_{eff} (in MeV/fiss.) that correspond to the midpoint of the reactor operating period:

$$
\begin{aligned}
&^{235}\text{U} : 193.37 \pm 0.33 \qquad &&^{238}\text{U} : 194.60 \pm 0.81 \\
&^{239}\text{Pu} : 199.80 \pm 0.28 \qquad &&^{241}\text{Pu} : 202.04 \pm 0.32.
\end{aligned} \tag{16}
$$

3 TOTAL THERMAL ENERGY E_f

In this section, we present the results obtained by calculating the energy E_{nc} (in MeV/fiss.) absorbed in a reactor upon the capture of neutrons not involved in the chain reaction, determine the total thermal energy E_f, and consider its variation within the reactor operating period.

1. Of the total number n_f of neutrons emitted in a fission event, only one contributes to the chain reaction. The remaining neutrons are absorbed almost completely in the reactor core, reflector, and vessel. The probabilities of the absorption of these neutrons by various substances and the energies E_{nck} released in the capture of one neutron in those substances are quoted in Table 3.

Table 3: Balance of the absorption of neutrons not involved in the chain reaction and of the thermal energy E_{nck} released upon the absorption of a single neutron in a given material (midpoint of the operation period)

Material	Capture probability η_k,%	E_{nck}, $\frac{MeV}{neutron}$	Material	Capture probability η_k,%	E_{nck}, $\frac{MeV}{neutron}$
^{235}U	11.6	6.54	^{149}Sm	0.8	7.99
^{238}U	38.4	5.72	Other fragments	6.8	7.88
^{239}Pu	10.5	6.53	Zirconium	7.0	8.11
^{240}Pu	6.1	5.24	^{10}B	5.6	2.79
^{241}Pu	3.6	6.31	Water	4.4	2.22
^{135}Xe	3.4	7.49	Other materials	1.8	5.67

From those data, it can be seen that more than 80% of $(n_f - 1)$ neutrons are absorbed in the fuel and in the accumulated fission fragments. In all

cases, with the exception of that of ^{10}B, the neutrons are absorbed via (n, γ) reactions. The energies E_{nck} include the energy of photons emitted in radiative neutron capture and, if beta-radioactive nuclei are formed, the energy of beta electrons and photons originating from the subsequent transformations of these nuclei.

The mean energy absorbed in the reactor in the capture of one neutron, $E_{n1} = \sum \eta_k E_{nck}$, and calculated on the basis of the data given in Table 3 is $E_{n1} = 5.97 \pm 0.15$ MeV/neutron, its increase within the period from 1 day to the end of the operating period being 0.55 MeV/neutron.

At the midpoint of the reactor operating period, the energies $E_{nci} = E_{n1} \cdot (n_{fi} - 1)$ (in MeV/fiss.) entering into the total thermal energy of the fission of uranium and plutonium isotopes are:

$$
\begin{aligned}
&{}^{235}\mathrm{U} : 8.55 \pm 0.22 \qquad &&{}^{238}\mathrm{U} : 10.92 \pm 0.28 \\
&{}^{239}\mathrm{Pu} : 11.19 \pm 0.28 \qquad &&{}^{241}\mathrm{Pu} : 11.56 \pm 0.29.
\end{aligned} \tag{17}
$$

2. We now present the values obtained for the total thermal energies E_{fi} of fissile isotopes by summing the components found above, see Table 4.

Table 4: Thermal fission energies E_{fi}, at the midpoint of the reactor operating period

Isotope	E_{fi}, MeV/fission
^{235}U	201.92 ± 0.46
^{238}U	205.52 ± 0.96
^{239}Pu	209.99 ± 0.60
^{241}Pu	213.60 ± 0.65

The total thermal energy $E_f = \sum \alpha_i E_{fi}$ and the contributions of fissile isotopes to the total number of fission events within the operating period of a PWR reactor are given in Fig. 4 versus the time of reactor operation. At the midpoint of the operating period, we have $E_f = 205.3$ MeV/fission. The errors in the values E_f are estimated at 0.6 MeV, which corresponds to about 0.3%. They include both the errors in E_{tot}, $\langle E_\nu \rangle$, $\Delta E_{\beta\gamma}$, and E_{nc} and the errors in α_i. It is assumed that the latter are 5% (relative errors). The increase in E_f over the segment from 0.5 d after the start-up to the end of the operating period is 3.75 MeV. This increase is caused by three reasons:

the growth of the energy E_{nc} released in neutron capture, a decrease in the fraction of ^{235}U and an increase in the contributions of ^{239}Pu and ^{241}Pu in the process of reactor operation, and the start-up effect that is associated with the growth of the beta- and gamma-radiation energy and which is the most sizable within the first week after the start-up (see Fig. 4).

4 CONCLUDING COMMENTS

The energy E_f, which relates the number of fission events occurring in a reactor to its thermal power, has been calculated with an error of $\delta E_f/E_f \approx 3 \times 10^{-3}$. The high precision of the calculation of this energy has been achieved owing to the possibility of finding its main component E_{tot} with a relative error as small as about $5 \cdot 10^{-4}$. The three other components, $\langle E_\nu \rangle$, $\Delta E_{\beta\gamma}$, and E_{nc}, have been computed to a poorer precision, but they are relatively small, not exceeding 5% of E_f .

The energy E_f increases throughout the operating period. At a constant thermal power, the number of fission events in the reactor decreases from the beginning to the end of the operating period.

We note that all of the components appearing in expression (3) for the energy E_f, with the exception of E_{nc}, are characteristics of the fission of the nuclei being considered, so that their calculation is based on nuclear-physics data − in particular, data associated with the physics of fission.

The special features of a reactor manifest themselves in the following:

- Use is made of the chain-reacting condition, which implies that one of the fission neutrons from the preceding generation induces one new fission process in the next generation.
- Numerical data on the fission branching fractions α_i and on their time dependence are employed.
- The term E_{nc} is calculated with the aid of data on the balance of neutron absorption in a reactor.

As a typical example, we have presented results (see Fig. 4) concerning a standard operating period of PWR reactors, which are widely used in Europe, the United States of America, and Japan. However, an actual operating period of a PWR reactor may differ from a standard one significantly. There also exist other high-power reactors at which neutrino investigations are being

presently performed or are planned. These reactors differ from their PWR counterparts by the duration of the operating period, the enrichment of the nuclear fuel used, and some other special features. In all such cases, the method developed in the present study and the results obtained here can be used in neutrino investigations to perform a quantitative analysis of the relationship between the level of power and the rate of the chain reaction in the reactor core.

For the first time, thermal fission energies were calculated more than 30 years ago [14]. Later on, a new calculation was performed [15] in connection with neutrino investigations at the Rovno atomic power plant. In the present study, we have employed the most recent data concerning the issue being considered and, for the first time, have traced the dynamics of thermal fission energy throughout the reactor operating period.

Acknowledgments

We are grateful to M.S. Yudkevich, V.D. Sidorenko, and S.N. Bolshagov for consultations on problems in the physics of nuclear reactors.

This work is supported by the Russian Foundation of Basic Research (project no. 03-02-16055) and was also funded with a grant in support of leading scientific schools.

References

[1] Yu. Kozlov et al., Nucl. Phys. B (Proc. Suppl.) **87**, 514 (2000).

[2] A.G. Beda, E.V. Demidova, A.S. Starostin, and M.B. Voloshin, Yad. Fiz. **61**, 72 (1998) [Phys. At. Nucl. **61**, 66 (1998)].

[3] M. Appolonio et al. (CHOOZ Collab., Phys. Lett. B **420**, 397 (1998); **466**, 415 (1999).

[4] KamLAND Collab., Phys. Rev. Lett. **90**, 021802 (2003).

[5] H.T. Wong and J. Li arXiv: hep-ex/0201001.

[6] S. Schoenert, T. Lassere, and L. Oberauer, hep-ex/0203013; Astropart. Phys., **18**, 565 (2003).

[7] A.V. Derbin, Fiz.@ Elem. Chastits At Yadra **32**, 739 (2001);
L.A. Mikaelyan, Yad. Fiz. **65**, 1206 (2002) [Phys. At. Nucl. **65**, 1173 (2002)].

[8] G. Audi and A.H. Wapstra, Nucl. Phys. A **595**, 409 (1995).

[9] T.R. England and B.F. Rider, LA-UR-94 3106, ENDF-349, LANL (Los Alamoos, 1994).

[10] L.P. Abagyan et al., Vopr. At. Nauki Tekh., Ser.: Fiz. At. Reaktorov **3**, 50 (2001).

[11] K. Schreckenbach et al., Phys. Lett. B **160**, 325 (1985); A. Hahn et al., Phys. Lett. B **218**, 385 (1989).

[12] V.I. Kopeikin, L.A. Mikaelyan, and V.V. Sinev, Yad. Fiz. **64**, 914 (2001) [Phys. At. Nucl. **64**, 849 (2001)].

[13] V.I. Kopeikin, L.A. Mikaelyan, and V.V. Sinev, Yad. Fiz. **60**, 230 (1997) [Phys. At. Nucl. **60**, 172 (1997)].

[14] M.F. James, J. Nucl. Energy **23**, 517 (1969).

[15] V.I. Kopeikin, Preprint IAE-4305/2 (Kurchatov Institute of Atomic Energy, Moscow, 1986).

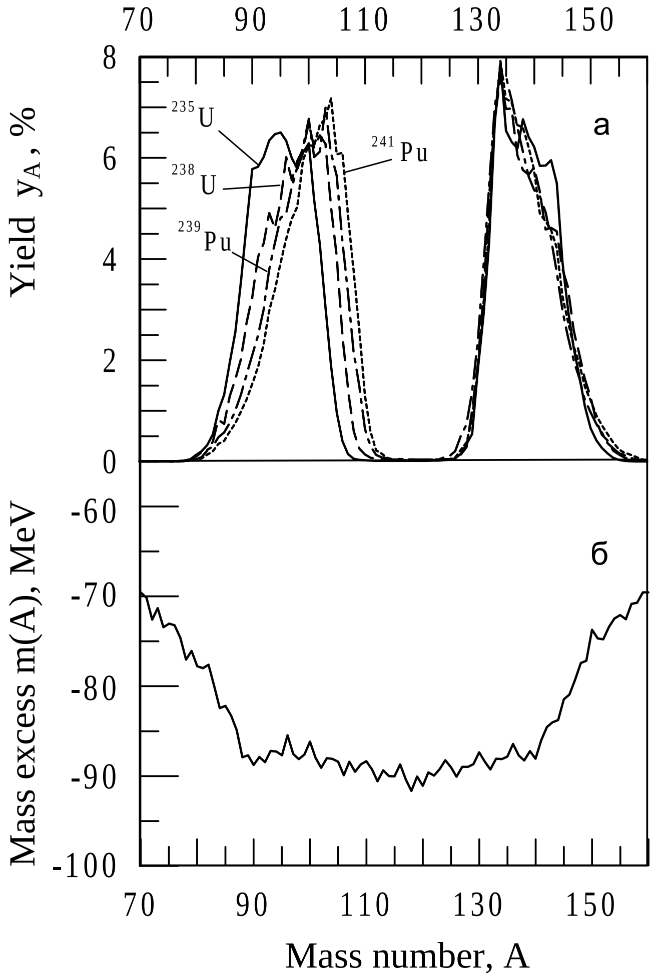

Fig. 1. (a) Total yield *y(A)* of beta-stable fragments originating from the fission of uranium and plutonium isotopes; (b) mass excess *m(A)* for beta-stable atoms as a function of the mass number *A*.

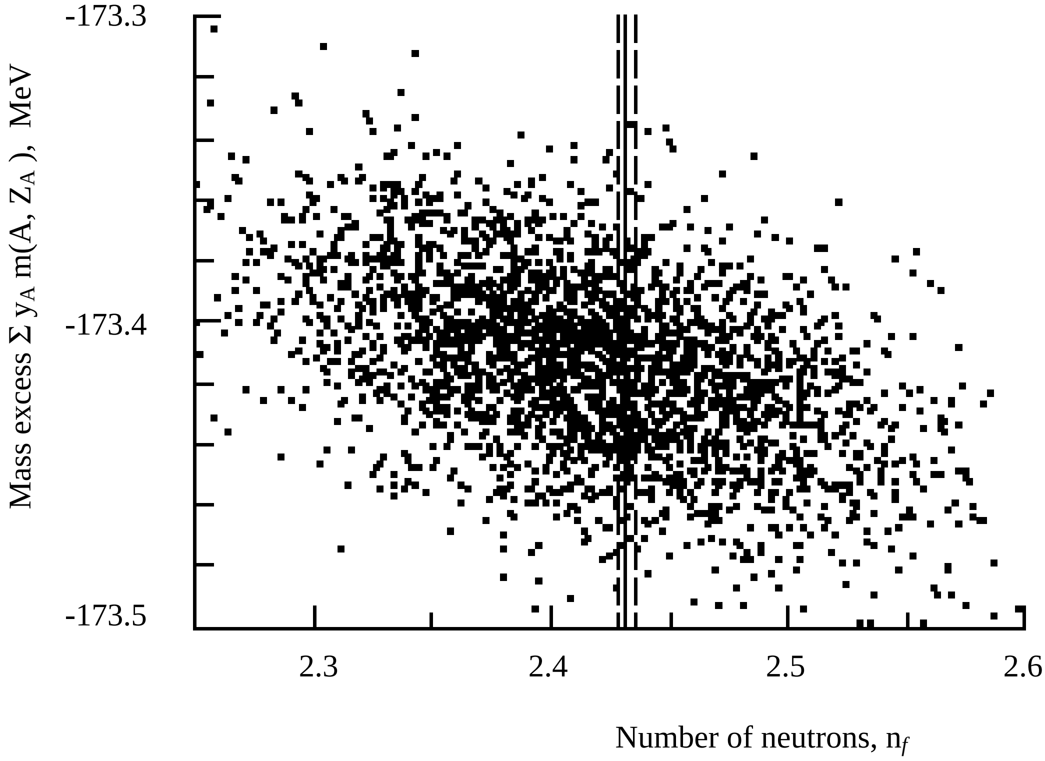

Fig. 2. Effect of the errors in the yield of the products originating from ^{235}U fission on the mass excess $\sum y_A m(A, Z_A)$ (see main body of the text). The vertical band corresponds to the experimental value of the number n_f of neutrons.

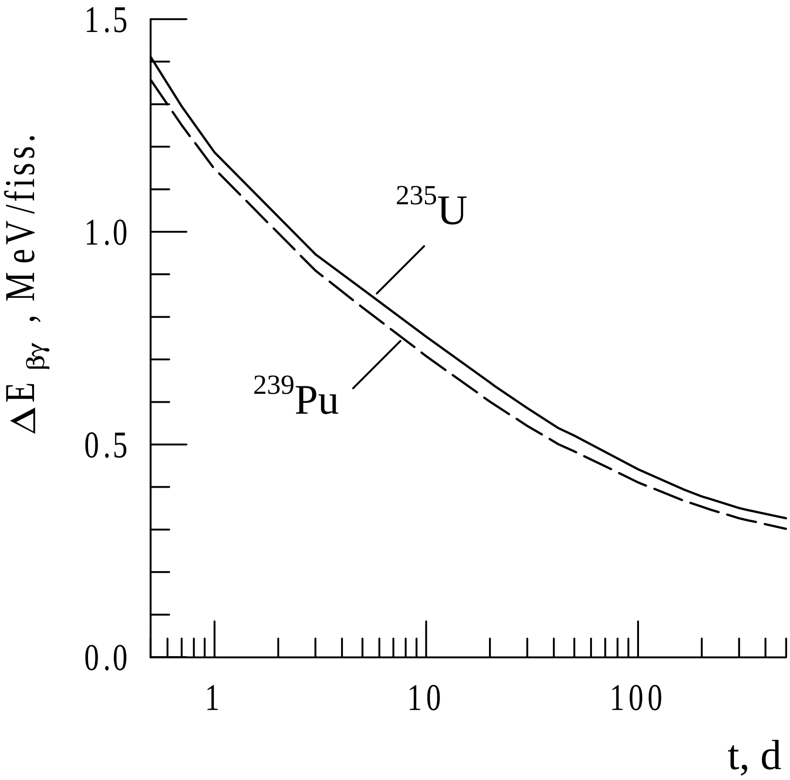

Fig. 3. Energy $\Delta E_{\beta\gamma}$ of beta and gamma radiation from fission fragments that did not decay as a function of the chain reaction time t.

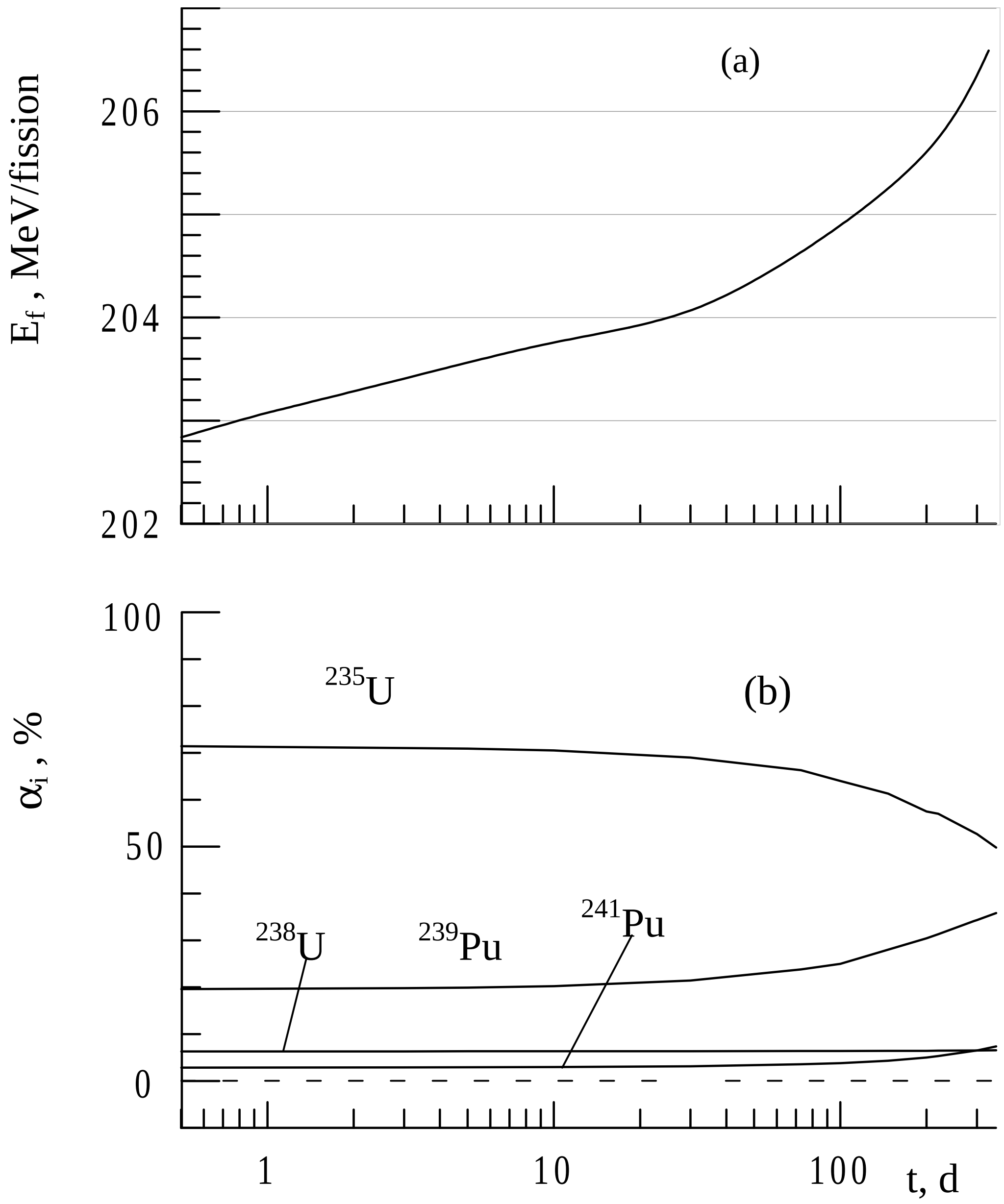

Fig. 4. (a) Total thermal energy E_f and (b) contributions α_i of fissile isotopes to the total number of fission events within the standard operation period of the PWR reactor versus the reactor-operation time *t*.